University of Oslo / Department of Economics

ECON3120/4120 Mathematics 2: On the 2024-12-06 exam

- *Standard disclaimer:* A note like this is not suited as a template for an exam paper. It was written as guidance for the grading process – however, with additional notes and remarks for using the document in teaching later.
  - The document reflects what was expected in that particular semester, and which may not be applicable to future semesters. In particular, what tests one is required to perform before answering «no conclusion» may not apply for later.[*]
- Weighting: Problem set *suggested* uniform weighting over letter-enumerated items, leaving a decision up to the graders (and in case of appeals: the new grading committee).
- Format and resources available: Being done with the COVID format, we are returning to what was introduced in 2019. Exam is four hours, a quite extensive collection of rules and formulae available on-screen together with the problem set – as well as a simple (four functions and square root) calculator. Also, the exam is only available in English.
- The 2019 exam format change was not intended to change overall requirements. Rather, it was intended to facilitate better differentiation between candidates.
  - Grading scale had been defaulting to thresholds 91–75–55–45–40 more or less since the ECTS letter grades were introduced. Upon the change in exam format in 2019, a slightly easier set was intended to facilitate the 92–77–58–46–40 scale once recommended by the Norwegian Mathematical Council and commonly applied at The Faculty of Mathematics and Natural Sciences.
  - It was a hope that the effort put into the 2019 grading would set a practice[†] – but as it turned out, the next exam had the COVID format. It would be a stretch to say that one *established* any change in grading scale.
- 2019 also saw a major change in compulsory activities (with slight revision in 2022). More problems were made compulsory than in the age of 3h exams, and one could no longer bring the problems and solutions to the exam; thus making it possible to align parts of the exam more closely up to parts of the compulsory problem sets.

[*] The standard disclaimer was kept even if the 2023-12-18 set has no such classification problem that could lead to a «no conclusion» answer.

[†] Quoting from the 2019 grading guideline:
« Given that this is the first exam set in the new format, it might set standards for the years to come, and the committee should set benchmarks with caution. There might be less reason to stress the percentage-to-grade tables that have been applied earlier (nominally defaulting to 91–75–55–45–40); the 40 percent pass mark does however hold a long history and as a preliminary view I would consider it to be more of a constant than the other thresholds. »
The 2019 exam(s) did end up applying 92–77–58–46–40.

New in 2024: Significant syllabus changes. Impact on the exam:

- The formula collection[‡] had to be revised. It was attempted to keep the changes down to what was called for. Up to minor fixes in text and typography, sections ABCGH are unchanged, and also the majority of the other four sections.
- On Problem 4: As inequality constraints are no longer syllabus, it takes three variables to test knowledge of how to even handle two constraints. That does pose a challenge for sufficient conditions, as trivariate concavity is not syllabus – except, the concavity of a sum of concaves is.
- Integration and differential equations syllabus is reduced. Impact on Problem 5.
- Then the new topics: difference equations (start of Problem 5), and dynamic optimization (Problem 3).
- In a broader picture, this class has not had too many complete old exam problem sets that are all within syllabus – and none that cover the new topics anyway. Therefore, the last two of the seminar problems were crafted to look more like what an exam could have looked like – and those sets are likely those which look most alike this exam. Contrast this to e.g. how the 2019 solution note likens that set to the hand-ins that were given earlier in the semester.

On this set. Due to the latter bullet items, the set has very much in common with the penultimate and final seminar set, so larger parts of it should be both familiar and fresh in mind. And, given the changes, we attempted not to put in too many «nuts to crack» this time. All this, we think, makes the set not too hard even with new syllabus.

On the other side of the coin, the set was then not on the short side; some of the questions can be answered in a couple of lines, but others can definitely not.

Hopefully this strikes a good balance. This document will be revised post-grading with actual grading thresholds applied.

POST-GRADING UPDATE: This problem set was too long, and the grading thresholds were reduced to 88–72–53–40–35. Problem-by-problem post-grading remarks in footnotes next pages!

Rest of document:

- Remarks to each problem. Includes seminar problems to relate them to (unlike previous years, this guide note does *not* include the actual exam questions). *All footnotes are post-grading remarks.*
- Nils' handwritten solution (*caution: quite terse,* as other handwritten solutions available).
- Herman's typeset solution, more elaborate.

[‡] It should be noted that citing the formula collection as source is surely allowed – but one needs to ensure that the formula cited covers the attempted application. Identifying the problem type is a big part of this course.

Remarks to problem 1 [1] This is a common problem type, although it was not featured in the 2021 exam. The following was assigned for the final seminar, and was itself an adaptation to exam 2023 problem 1; it has the additional complicating layer that it asks to use the derivative to get a first-order approximated value.

This problem was assigned for sem#13

(a) Variable-by-variable is OK, term-by-term is OK even if they are not reordered afterwards, like this solution by Wolfram Alpha. Some reordering has to be done in (b) anyway.

(b) The right method is key. It is likely possible to get «A» score with wrong answer.

[1] Post-grading remarks:
(This problem came about by taking $Q(K, L)$ as Cobb–Douglas + square root functions, differentiating $Q(K, L) - pK - 7L$, putting equal to zero, and multiplying by K resp. L.)

(a) was very well answered – seems to average «A» when the failing papers are removed – although several seem to think that the derivative of $3(KL)^{1/3}$ is $(KL)^{-2/3}$ – chain rules, please!

(b) averaged around the pass mark although generous score was given on wrong answers with right method. Common mistakes that are *not* within «right method», include zeroing out the wrong variables like dp or dK.
Less serious, except being costly on time: It was striking how many who skipped the simplifications available, or were unable to go from $64 = 4^3$ to $64^{1/3} = 4$. For example, a paper would drag $16 \cdot 64^{-2/3}$ through the calculations rather than simplifying it to 1 – on top of a problem set with a high workload.

Remarks to problem 2 [2] Has quite a lot in common with 2019 problem 2, which was assigned in the final «stumble group» problem set. The 2019 solution note restates the problem and explains why such problems are given.

Parts of it – testing how to multiply matrices and calculate determinants – were also found in exam 2023 problem 2, that was assigned for the final seminar. The 2023 solution note is here.

Some remarks:

(a) Stating outright that there is a square matrix that is not 4×4, maybe gives away the fact that matrix multiplication is not done element-wise.

(b) «Up to eight» ... it is three, and $|A_t^2| = |A_t|^2$, it was deliberate that they do not *need* to calculate the matrix product. But one cannot deny full score if that was the method.

(c) Hopefully, those who know will spot that they should use $|A_t|$.

[2] Post-grading remarks: Problem 2 seems to be the highest-averaging of the five, with the first two parts achieving good answers in *most* papers. However, many papers did long and elaborate answers, which did not help for a slightly long set.

(a) was scored very high – although the graders did not calculate rigorously after the grading thresholds were adjusted, it might be that the average *passing* paper's score was within the «A».

(b) scored lesser but still good – at least «B» level among the passing. As expected, several got signs in cofactor expansions wrong. Also, a sizeable $3(t + 4)$ had wrongly became either $3t + 4$ or $t + 12$. Also as expected: many had to resort to calculating A^2 rather than using $|A^2| = |A|^2$.

(c) scored quite bad, although still «C» level if the failing papers were removed. A frightening number of scorepoints were lost from ignoring the problem text that outright says that $(1, 0, 0, 0)$ is a solution – so why then make claims that no solution exists? Mistakes noted:

- The stubbornly widespread false equivalence between nonzero determinant and *existence*. Remember: $0\mathbf{x} = \mathbf{0}$ – or even the special case of one variable, $0x = 0$. (Though should be added: part of it must be failure to understand the concept of logical equivalence. It is easier to make wrong «if and only if» claims if you do not know what «if and only if» *means*.)
- Failure to read what a matrix equation means. A row $(0\ 0\ 0\ 0 \,\vdots\, 1)$ means $0 = 1$ and no solution. A row $(0\ 1\ 0\ 0 \,\vdots\, 0)$ means that $x_2 = 0$, and that is something completely different.
- Confusing the condition that the *determinant* be zero, with t being zero.

Remarks to problem 3 [3] The following was assigned for the final seminar:

This problem was assigned for sem#13

The magenta-colour text was added after it was published, because a hint is arguably due; Hessian-based criteria for > 2 variables is *not syllabus.* The reason why the hint was given as a reference to the formula collection, was to make aware that it has such criteria too – in case one would need it on the exam. (Which was the intention, but that wasn't communicated.)

---

[3]Post-grading remarks:

This problem has several «Math 1» elements with disappointingly weak answers.

(a) was one of the top four scoring parts, together with 1(a) and 2(a) and (b). We would have wished for more papers to include a «> 0» after the second derivatives. Yes we know that 6 > 0 – but is it obvious that you know the sign is a key property, not only for the Hessian? Some papers were evidently wishfully thinking that every term was convex – even $-xy$ or xy (ignoring the sign).

(b) not bad at all among the passing papers, but exposed a whole lot of weaknesses, including not knowing that Lagrange conditions include stationarity of the Lagrangian. We could see mistakes like the following:

  i) Checking merely admissibility, not the Lagrange conditions.

  ii) Then only in this part doing anything about the Lagrange conditions. But, not necessarily verifying them: just calculating one multiplier from L_y and the other from another equation, not checking the third one.
  That makes it hard to judge several papers that *did* insert in all three – was that an attempt at checking all three, or just to see what was the best strategy to extract the multipliers? Put on top of this that the minus sign in the second constraint often disappeared.

  iii) Now, claiming that yes we have shown that the Lagrange conditions hold – with no mention of convexity. (It was actually easier to believe that those who claimed *concavity* knew their sufficient conditions, since there are so many maximization problems.)

(c) had widely varying grades, and a sizeable number of passing papers got zero score – even several «B» papers. Although the second question here must be considered tricky, we firmly believe that master-level economics students should know that the Lagrange multiplier is the shadow value/price on the constraint. Yet very few pointed it out.

(a) Again, there should be a hint. This is hopefully clear enough.

(b) This asks quite a lot of elements, but asking for this sort of detail hopefully makes it easier for graders to assess whether they have actually checked that both multipliers fit all three first-order conditions.

(c) The second of these is less familiar than just pointing to the multiplier; here one needs «more of the envelope theorem».

Remarks to problem 4 [4] New topic, no previous exam paper to judge from, so: starting out «nearly as kind as possible» (though yes it could be without discounting. The H variable isn't really a complication: it makes it easy to recycle the calculations for the $T - 2$ step.)

This problem was assigned for sem#12

... the footnote saying that it could be enough work to warrant «two letter-enumerated items» weight. On the exam, it was split parts (a) and (b), but it is up to the graders to ultimately decide on weighting. Problems assigned as part of hand-in 4, were a bit different:

[4] Post-grading remarks: Problem 4 overall had the weakest score among the five.

(a) averaged in the «D» ballpark although «C» among the passing papers. And exhibited several issues: when there is a discount factor of 1/2 – maybe not the realistic one, but at least on the reasonable side of 1 – then one should not need to spend that much ink going from $(1/2)^{T-(T-1)}$ to the wrong «2». Instead the dynamic programming equation allows to write down the 1/2 in front of next period's value function without going through those motions. Worse are the differentiation skills. $\frac{d}{dc}(x - c)^2$ had several creative versions. The «2» was lost, the minus sign was lost, even the differentiation itself was lost. And when the first-order condition is claimed to be $0 = c^2 + (x - c)^2 \cdot$(a positive constant), it does require some sign error or worse to get a real-valued c out.

(b) scored below the pass mark . Of course, several papers that could get away with (a) without properly using the dynamic programming equation, got in all sorts of problems here. But surprisingly many who found a correct or incorrect c_{T-1} inserted it, got a sum of squares, and failed to notice that it is proportional to x^2 with whatever constant in front. Although several questions do not require any rewriting to get answers on a nicer form, this v_{T-1} is something needed for further calculations.

For both (e) and (f): You are required to write out the calculations rather than fetching answers from page 12 of the dynamic optimization note.

Problem (e) is supposed to be not too different from the one in section CS.1. You may take note that 64/125 equals $(4/5)^3$.

Task: Calculate the optimal c_{T-2} at $t_0 = T - 2$ for the dynamic programming problem

$$v_{t_0}(x) = \max_{c_t \in (0,1)} \left[(64/125)^{T-t_0} \cdot \frac{x_T^{-2}}{-2} + \sum_{t=t_0}^{T-1} (64/125)^{t-t_0} \cdot \frac{(c_t x_t)^{-2}}{-2} \right] \qquad \text{(E)}$$
$$\text{where } x_{t+1} = (1 - c_t)x_t, \quad x_{t_0} = x \quad (\text{given}, > 0)$$

Problem (f) is supposed to be not too different from the one in section CS.2. *Mind the «2» in $x_{t+1} = 2(1 - c_t)x_t$!*

Consider the dynamic programming problem

$$v_{t_0}(x) = \max_{c_t \in (0,1)} \left[\left(9/10\right)^{T-t_0} \cdot 3x_T^{2/3} + \sum_{t=t_0}^{T-1} \left(9/10\right)^{t-t_0} \cdot \frac{3(c_t x_t)^{2/3}}{2} \right] \qquad \text{(F)}$$
$$\text{where } x_{t+1} = 2(1 - c_t)x_t, \quad x_{t_0} = x \quad (\text{given}, > 0)$$

Take for granted the fact that $v_{T-35}(x) = \frac{3}{2}Ax^{2/3}$, for some constant $A > 0$.
Task: Calculate $v_{T-36}(x)$ (in terms of A).

These problems assigned for hand-in 4

Remarks to problem 5 [5] Again, new topic with no previous exam paper to practice on – but some assignments. 3 of 7 parts of hand-in 3:

(a) For the difference equation $x_{t+1} = \frac{3}{4}x_t + 1$:
- Find the general solution.
- Find the particular solution which satisfies $x_{2024} = 3$.

(b) Show that $x_t = (-1)^t(c + t)$ solves the difference equation $x_{t+1} = -x_t - (-1)^t$ with $x_0 = c$

(c) Consider the difference equation $x_{t+1} = -\frac{1}{2}x_t + e^t - (\pi/2)^t$.
- Find a particular solution of the form $\alpha \cdot e^t + \beta \cdot (\pi/2)^t$.
- Find the general solution.

These problems assigned for hand-in 3

From some apparent confusion on ambition level, it was announced that particular solutions

[5] Post-grading remarks: for layout, footnote continues last page.

with e.g. «x_{2024}» given, are not to be given on the 2024 exam (that is not a promise for 2025!) – reflected in the «when $t = 0$» in the next seminar problem:

This problem was assigned for sem#12

... while the final seminar set does not cover any given initial value:

This problem was assigned for sem#13

To each letter-enumerated part:

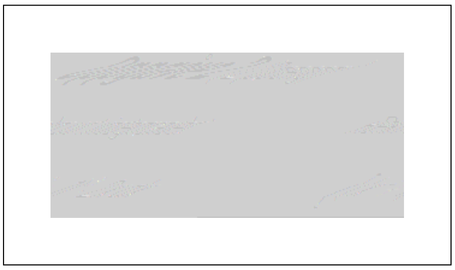
(a) Pretty well aligned to seminar 12 problem 3, but doesn't require to distinguish between $-m^t$ and $(-m)^t$.

(b) «Experiment:» How many choose (D) and how many choose (E)?

(c) This is the last question on hand-in 2 – with the same letters z, r, α_r, β_r, γ_r – although with the «Note» added.

(d) Only chance to assign this, before the course is attended by those who have seen it on their exam just before being admitted to the 5-year program with «S2» math from high school:

From the S2 exam spring 2024



Integration by substitution is no longer syllabus in Math 2, hence the wording. We expect «Expand the polynomial!» as answer. But, also the following should get full score, and is also more in line with a question on hand-in 2, that asked to explain how to calculate $\int w^{2024}e^{rw}dw$:

> «Integration by parts with $u = (y^2+1)^3$ and $v' = 2y$ leads to the problem of integrating $3(y^2+1)^2 \cdot 2y \cdot y^2 = 6(y^2+1)^2 \cdot y^3$. Then integration by parts again.»

(This is *more than* sufficient. It is surely not necessary to write out «again *and again*» or indicating how many times. Arguably, «integration by parts, differentiating down the $(y^2+1)^3$» could do – except, the chain rule could make for a surprise that fortunately doesn't ruin the attempt.)

---

[5]post-grading note 5 cont'd:

Problem 5 did not score too well, yet still «C» among the passing papers. Overall length may be a factor, but several issues are due to conceptual mistakes.

(a) had several mistakes.
- Simple ones, like $2^{-(t+1)}$ getting the parentheses lost in the next step.
- Differentiation mistakes, in particular using the t^n rule rather than the a^t rule.
- Conceptual: Failure to realize what a *constant* is. Often by randomly matching constant $\times\, q \cdot 2^{-t}$ to 7 and constant $\times h$ to -2^{-t} and similar for (E).

(b) scored at least not much worse. And a rough overview indicates that the new topic of difference equations wasn't eschewed. Mistakes included (for (E)): putting $t = 0$ both outside and inside the integral, getting $Ce^0 + e^0 \int (7 - e^{-0})dt$ and from then on concluding that the *constant* C would have to be $-6t$ – or, that $C - 6t + C$ must be 19.

(c) Lots of long calculations to get the constants out explicitly (that was not asked for), and simple calculations mistakes were graded generously, ignoring omissions of case $r = 1$. Worse are those papers that failed at $\frac{d}{dz}(\ln z)^2$. Reminder that the chain rule for differentiation also made for trouble elsewhere.

(d) At the end of a long set, this was a hit-or-miss – but that must be expected from the problem formulation. And among those with an answer: Several paper tried to state something in the vein of «using integration by parts» without indicating anything how that could be helpful (yes there is a way to make it work, but how?). What to differentiate down? Several arguments that are really substitution. And no, $(y^2+1)^3$ is not equal to $(y^2)^3 + 1^3$.